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BOOK OF ABSTRACTS

Euro-American Consortium for Promoting the Application of Mathematics in Technical and Natural Sciences

Injectivity and Exact Inversion of Ultrasound Operators in the Spherical Geometry

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In ultrasound tomography an emitter sends acoustic waves through the body, and the reflections of these waves are registered by a receiver. These data measured for various locations of emitter and receiver are then used to reconstruct the acoustic reflectivity function, which represents an image of the interior of the body. Mathematically this procedure is equivalent to the inversion of an operator, which puts into correspondence to the image function the measured reflections at available receiver locations. The talk discusses the injectivity of ultrasound operators in the spherical geometry of data acquisition, and exact inversion procedures derived for several setups in this geometry.

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State Estimation for Linear Stochastic Differential Equations with Uncertain Disturbances via BSDE Approach

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A backward stochastic differential equation (BSDE) is an Ito stochastic differential equation (SDE) for which a random terminal condition on the state has been specified. The linear version of this type of equation was first introduced by J. Bismut as the adjoint equation in the stochastic maximum principle. BSDEs have received considerable research attention in recent years due to their nice structure and wide applicability in a number of different areas, especially in mathematical finance. The talk deals with estimation problems for partly observed stochastic processes described by linear SDEs with uncertain disturbances. The disturbances and unknown initial states are supposed to be constrained by the inequality including mathematical expectation of the integral quadratic cost. On the assumption of measurability of the disturbances with respect to the sigma-algebra generated by the observable part of the process, we use the nonlinear filtering equations by R. Liptser and A. Shiryaeyv, consider them as BSDEs, and construct at given instant the random information set of all possible states which are compatible with the measurements and the constraints. The center of this set represents the best estimation of the unobservable part of the process. The evolutionary equations for the random information set and for the best estimation are given and the properties of these equations are examined. The solution of the problems obtained completely and explicitly by using an approach which is based primarily on the completion-of-squares technique elaborated by A. Lim and X. Zhou. A comparison with purely deterministic estimation problem considered by A. Kurzhansky, R. Mortensen, W. McEneaney, and others is fulfilled. Some examples and applications to navigation and finance are considered.

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Hausdorff Continuous Solutions of Conservation Laws

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We present a framework in which to study (generalized) solutions of the Cauchy problem for nonlinear conservation laws, in particular entropy solutions of scalar conservation laws. The space of generalized functions are constructed as the completion of the space of continuously differentiable functions with respect to a suitable uniform convergence structure. Within this context the well-posedness of the problem follows in an easy and natural way. Furthermore, we show that the space of solutions is a subspace of the space of Hausdorff continuous interval valued functions which improves significantly on the current regularity results of the entropy solution.

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Dimensionless Goodwin's Equation of Business Cycle

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We have considered the Goodwin model of the economic cycle [1]

$$\varepsilon\frac{dy(t)}{dt} + (1-\alpha)y(t) = \varphi\Big(\frac{dy(t-\theta)}{dt}, r, \varphi_c, \varphi_f\Big).$$

Here $y(t)$ is income, $\varepsilon > 0$ and $\theta > 0$ are the time-lag of the dynamic multiplier and the time-lag between investment decisions and the resulting outlays, respectively, α is the marginal propensity to consume, $0 \leq \alpha \leq 1$. φ is the investment induced by the change in income, where r is the acceleration coefficient,φ_c and φ_f are the Hicksian ceiling and floor. Solutions of this equation have the form of the long-period Goodwin's oscillations or the short periodic sawtooth oscillations [2].

The above equation contains six parameters. We have shown it has been conveniently written in a dimensionless form

$$\frac{du}{d\tau} + u = \varphi\Big(\frac{du(\tau-\theta_8)}{d\tau}, r_*, k\Big),$$

which contains only three parameters

$$r_* = \frac{r}{\varepsilon}, \qquad \theta_* = \frac{(1-\alpha)\theta}{\varepsilon}, \qquad k = \frac{\varphi_c}{|\varphi_f|}.$$

The dependence of oscillation period and amplitude on the parameters values r_*, θ_* and k_* has been numerically investigated.

References

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Efficient Implementation of a Stochastic Electron Transport Simulation Algorithm Using GPGPU Computing

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The modeling of electrons stochastically is important. Monte Carlo and quasi-Monte Carlo algorithms as well as hybrid algorithms are applicable. GPGPU computing emerged as an effective way of performing Monte Carlo type simulations, exploiting fully the inherent fine-grain parallelism. In this work we describe our new implementation of an algorithm for semiconductor device modeling, using NVIDIA CUDA. Numerical and timing results are presented, outlining the advantages of the algorithm compared to previous experiments performed on HPC clusters and Blue Gene/P supercomputer. We analyse the performance of the algorithm and the issues related to the efficient use of GPGPU computing in this case.

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Numerical Simulation of Drop Coalescence in the Presence of Film Soluble Surfactant

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Numerical method is presented for simulation of the deformation, drainage and rupture of axisymmetrical film (gap) between colliding drops in the presence of film soluble surfactants under the influence of van der Waals forces at small capillary and Reynolds numbers and small surfactant concentrations. The mathematical model is based on the lubrication equations in the gap between drops and the creeping flow approximation of Navier-Stokes equations in the drops, coupled with velocity and stress boundary conditions at the interfaces. A nonuniform surfactant concentration on the interfaces, related with that in the film, leads to a gradient of the interfacial tension which in turn leads to additional tangential stress on the interfaces (Marangoni effects). Both film and interface surfactant concentrations, related via adsorption isotherm, are governed by a convection?diffusion equation. The numerical method consists of: Boundary integral method for the flow in the drops; Finite difference method for the flow in the gap, the position of

the interfaces and the surfactant concentration on the interfaces; Finite difference method for the surfactant concentration in the gap. Second order approximation of the spatial terms on adaptive nonuniform mesh is constructed in combination with Euler explicit scheme for the time discretization. Tests and comparisons are performed to show the accuracy and stability of the presented numerical method.

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Computational Approach to Thornley's Problem by Bivariate Operational Calculus

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Thornley's problem is an initial-boundary value problem with a nonlocal boundary condition for linear one-dimensional reaction-diffusion equation, used as a mathematical model of spiral phyllotaxis in botany. Applying a bivariate operational calculus we find explicit representation of the solution, containing two convolution products of special solutions and the arbitrary initial and boundary functions. We use a non-classical convolution with respect to the space variable, extending in this way the classical Duhamel principle. The special solutions involved are represented in the form of fast convergent series. Numerical examples are considered to show the application of the present technique and to analyze the character of the solution.

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On the Choice of Limiters in a Numerical Approximation of a Chemotaxis System with Non-homogeneous Boundary Conditions

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A mathematical model for the chemotactic movement of haematopoietic stem cells (HSCs) is proposed by A. Kettemann & M. Neuss-Radu (2008). It consists of a nonlinear system of chemotaxis equations coupled with an ordinary differential equation on the boundary of the domain in the presence of non-homogeneous boundary conditions.

Various classical numerical methods applied directly to this system may lead to numerical instabilities and loss of the positivity property of the solution. A finite-volume method, based on a second-order positivity preserving central- upwind scheme is proposed by A. Chertock & A. Kurganov (2008) for a class of chemotaxis and haptotaxis models with homogeneous Neumann conditions. The approach of A. Gerisch & M. A. J. Chaplain (2006) is also based on finite volume method for the case of zero-flux conditions. To ensure positivity and nonoscillatory nature of the numerical solution, minmod limiter is used by Chertock and Kurganov, while van Leer and Koren limiters are used by Gerisch and Chaplain for approximation of the first order derivatives of the unknown functions.

The approach of Chertock & Kurganov with appropriate modifications is applied in the current work for space discretization of the HSCs migration model. The nonlinear boundary conditions influence the resulting semi-discrete system from one side through the integrals on the boundary cells and from the other through the evaluation of the limiters, which are also nonlinear functions. The minmod, van Leer and Koren limiters are used and their influence on the properties of the numerical solution is theoretically and experimentally analyzed.

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Block Preconditioners for Mixed and Multiphysics Problems

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In many problems, we can meet a natural block structure. An example is e.g. mixed formulation of Darcy's problem with blocks corresponding to fluxes and pressures or poroelasticity where the blocks correspond to displacements, fluxes and pressures if the elasticity is expressed in displacement and Darcy's flow uses mixed formulation. The block systems can be solved iteratively by some Krylov space solver, in our case with the aid of MINRES and GMRES methods. An efficient iterative solution needs preconditioners, which can be built from the diagonal blocks and Schur complements. The smaller problems are solved by inner iterative solvers like conjugate gradients. There is a big number of variants of preconditioners, which differ in selection of pivot blocks and Schur complement on the top level, then differ in simplification of the Schur complement, and finally in the used inner solvers. These choices influence both efficiency of the method and capability of parallelization. In our contribution we will describe the variants and our numerical experience with various preconditioners. The efficient solvers are important with respect to solution of large scale problems of biomechanics and geomechanics as well as for application of parameter identification procedures in Darcy's flow and poroelasticity.

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BifTools: Maple Package for Bifurcation Analysis of Dynamical Systems

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Nonlinear dynamical systems depending on parameters may have very complicated behavior when some parameters are changed. These changes may include the appearance or disappearance of equilibrium points, periodic orbits or more complicated features. The center manifold theory proposes a systematic way for studying such kind of problems by simplification of the dynamical systems. Reducing a dynamical system to its center manifold means computation of a parameterdependent normal form for each bifurcation point in the minimal possible phase space dimension and specification of the relevant genericity conditions. The parameter and coordinate transformations required to put the system into the normal form lead to lengthy intermediate calculations for manual work like symbolic Jacobian computations, Taylor series coefficients, eigenvalues and eigenvectors computations. The natural environment for such kind of work are the computer algebra systems (CAS) like Maple and Mathematica.

We present the Maple package “BifTools” for symbolic study of one and two parameter local bifurcations of equilibrium points. The package is applied to investigating models of biological processes.

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Viscous Drag in Steady State Gas Flow between Rotating Cylinders

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The viscous drag exerted at the interface “gas-rotating cylinder wall” is numerically studied by using the Direct Simulation Monte Carlo (DSMC) method and the continuum model based on the Navier-Stokes-Fourier (NSF) equations for simulation of the cylindrical rarefied gas flow between two concentric cylinders with one of them rotating with a constant velocity and the other being at rest. Different cases were calculated for a set of velocities of rotating inner cylinder and another set for rotating outer one. These studies have been accomplished for several fixed Knudsen numbers. The NSF results have been obtained by setting a local value of Knudsen number in the corresponding first order slip boundary conditions. For the near planar case a typical averaged Knudsen number has been used. The flow characteristics obtained by both methods are: in an excellent agreement at small Knudsen numbers 0.02 and 0.05 and in a satisfactory agreement at 0.1 and 0.5. The work may be of interest to the analysis of various non-planar micro gas flows.

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Numerical Study of Fluxon Lattices in Long Josephson Stacks

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Magnetic field, applied along the layer direction of a long Josephson stack, may generate Josephson vortices in the junctions, resulting in a Josephson vortex (fluxon) lattices. We investigate numerically the dynamics of such lattices when increasing the external magnetic field, especially the transitions from in-phase motion to anti-phase motion and vice versa. Recently special attention is given to the experimental and numerical study of such fluxon motions because they are important for the applications. The investigation is made for different geometric and physical parameters of N stacked Josephson junctions. The inductive coupling model of the stack – a system of perturbed sine-Gordon equations – is used. The boundary conditions correspond to stacks of linear (open ends) geometry. Finite difference methods are used to solve the initial-boundary value problem for the system of nonlinear hyperbolic equations.

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Improved Algorithm for Modeling Inelastic Backscattering from Surfaces

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The problem for the electron scattering from targets is part of a major branch in the theory of studying and analyzing their quantitative and qualitative characteristics. In this paper we extend and enhance our Monte Carlo algorithm for modeling electron backscattering from surfaces to take into account the inelastic backscattering and to produce the whole reflection electron energy loss spectra (REELS). The new algorithm employs several variance reduction techniques in order to decrease computational time and has flexibility to simulate different physical models for the interactions.

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On the Axiomatization of the Scale-space Theory

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Scale-space is a common method for analysis of images. It uses an operator depending on a parameter, called a scale, which acts in some space of real functions. Many different scale-spaces have been defined and used in practice. The most popular example is the Gaussian Scale-Space where the operator is defined as an integral involving the Gaussian distribution function with the standard variation being the scale parameter. There have been also many attempts for axiomatization of the scale space theory. The novelty of our approach is that it does not assume integral representation of the scale space operator. While the new theory is applicable to and in fact represents and abstraction of the properties of all existing operators, it uses as an essential example the operator and the respective scale space provided by the discrete pulse transform [1].

References

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Parallelization of the Preconditioned IDR Solver for Modern Multicore Computer Systems

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In this work we perform the analysis of optimization and parallelization opportunities of the large sparse matrix solver CNSPACK for modern multicore microprocessors. CNSPACK is an advanced solver used for coupled solution of stiff. It employs iterative IDR algorithm with ILU preconditioning (of arbitrary chosen order). CNSPACK has been successfully used during last decade for solving problems in several application areas, including fluid dynamics and semiconductor device simulation. Originally, CNSPACK was implemented and optimized for early sequential processors, keeping in mind their arithmetic and memory-size limitations. CNSPACK performance with almost linear CPU time and memory versus N. In the beginning of 90's, the first optimization exercise was performed in order to accelerate (6X) the algorithm for appearing superscalar microprocessors (Intel i860). Current work discusses the approach leading to the linear speed-up versus number of cores.

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Numerical Study of the Wind Energy Potential in Bulgaria -– Some Preliminary Results

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The energy efficiency politics of the EU requires 11% till 2011 and 16% till 2020 of the Bulgarian electricity to be produced from renewable sources. An important aspect of the renewable energy is the reduction and the trade of CO_2 emissions, which determines its great ecological and economic importance and the need of better utilization of the country potential. That is why the renewable energy resources of the country, including wind energy, and their geographic pattern have to be well studied. Most of the planed and already build wind parks relay on the constant winds in the north-east part of Bulgarian coast. There are a lot of unexploited regions, however, which need further research so they can be attractive for investing in building big wind parks. Detailed study of the wind energy potential of the country – spatial distribution, temporal variation, mean and extreme values, fluctuations and statistical characteristics; evaluation from a point of view of industrial applicability is the objective of the present work. The wind field simulations were performed applying the 5th generation PSU/NCAR Mesometeorological Model MM5 for years 2000-2007 with a spatial resolution of 3 km over Bulgaria. Some preliminary evaluations of the country wind energy potential, based on the simulation output are demonstrated in the paper.

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High Fidelity Finite Length Markov Chain Walks

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Given an irreducible Markov chain model, what is the complete set of the highest fidelity state sequences of a given length (finite length walks), that the model is able to produce? Here, highest fidelity property means: 1) all sequences in the set have the same relative transition frequency matrix ; 2) according to some measure, the relative transition occurrence frequency matrix is as close to the model stochastic matrix as possible; 3) each sequence starts with a state that has nonzero initial probability. A three stage approach for generation of such complete set is discussed: 1) calculation of the exact absolute transition occurrence numbers matrix, which in general, consists of non-integer values; 2) application of controlled matrix rounding to the latter in order to obtain integer values for all transition occurrence numbers, which introduces a bias leading to the fidelity issue; 3) generation of all sequences that satisfy the constraints for state transition occurrence numbers from the integer matrix, which can be done by any constraint preserving permutation group generating technique. This paper is focused on the first stage an original linear algebraic solution to the problem is proposed. Three distinct and mutually complementary intuitive constraint aspects of the finite length Markov chain walks are formulated and formalized as three independent systems of linear equations with the exact absolute state transition occurrence numbers being the unknowns. Each system suffers from linear dependency among its equations. After proper elimination of certain part of each system and union of what is left of the three, one aggregated system is constructed. The conditions for the existence of such system's solution are investigated and its uniqueness is proven. The influence of solution's values over the existence of respective walks is considered. Several interesting properties of the system and its solution are discussed, some of them with proofs.

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Short Spirals on the Shape Sphere

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The simplicial shape space of order (2,3) is the set of equivalence classes of similar triangles in the plane. In 1984, D. Kendall introduced the first model of this quotient space, so-called the shape sphere. Nineteen years later, H. Nakamura and K. Oguiso obtained that the one-point extension of the Euclidean plane is another model of the same simplicial shape space. The present paper gives a description of a natural conformal mapping between two models. In this way we observe that the short spirals in the plane have analogues on the shape sphere.

We study properties of these spherical spirals in terms of special differential invariants called shape curvatures.

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A Class of Localized Solutions of the Linear and Nonlinear Wave Equations

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In linear regime the amplitude equation and wave equation discover non-paraxial semi-spherical diffraction of single-cycle pulses. In nonlinear regime the long (ns) pulses and the short (fs) ones admit different kind of nonlinearity and soliton solutions.

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Semi-coarsening Multilevel Preconditioning of Bilinear Nonconforming FEM Systems

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In this talk we will consider a nonconforming discretization of scalar elliptic problem. Different approaches for construction of two-level splittings of finite element space using semi-coarsening approach will be discussed. The constant in the strengthened CBS inequality indicates the quality of the splitting. Uniform estimates of the CBS constant with respect to the anisotropy ratio are derived. Combining this results with the theory of the hybrid V-cycle multilevel methods we obtain an optimal order multilevel preconditioner. Generalizations of the proposed approach to three dimensional problems will be discussed.

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Some Notes on Applying Approximate LU-factorizations as Preconditioners in Eight Iterative Methods for Solving Systems of Linear Algebraic Equations

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Many problems arising in different fields of science and engineering can be reduced, by applying some appropriate discretization, either to a system of linear algebraic equations or to a sequence of such systems. The solution of systems of linear algebraic equations is very often the most time-consuming part of the computational process during the treatment of the original problem, because these systems can be very large (containing up to many millions of equations). It is, therefore, important to select fast, robust and reliable methods for the solution of systems of linear algebraic equations when large applications are to be run, also in the case where fast modern computers are available. Since the coefficient matrices of the systems are normally sparse (i.e. most of their elements are zeros), the first

requirement is to exploit efficiently the sparsity. However, this is normally not sufficient when the systems are very large. The computation of preconditioners based on approximate LU-factorizations and their use in the efforts to increase further the efficiency of the calculations will be discussed in this paper. Computational experiments based on comprehensive comparisons of many numerical results that are obtained by using ten well-known methods for solving systems of linear algebraic equations (the direct Gaussian elimination and nine iterative methods) are reported in this paper. Most of the methods are preconditioned Krylov sub-space algorithms. The results from several hundred thousand tests indicate clearly that (a) the performance of the different Krylov sub-space iterative methods is similar when a sufficiently large set of test-matrices is used, (b) good preconditioners are needed and (c) it is essential to develop robust and reliable stopping criteria

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Tomographic Reconstruction of Harmonic Functions

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We consider an algebraic method for reconstruction of a harmonic function via a finite number of values of its Radon projections. More precisely, for given values of some Radon projections, we seek a harmonic polynomial which matches these data exactly. In the present work, we focus mostly on the case where these measurements are taken along equally spaced chords of the unit circle. We present an efficient reconstruction algorithm which is robust with respect to noise in the input data and provide numerical examples.

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Global Sensitivity Analysis of Compact Models in Nanodevices

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Statistical variability within transistors is a major obstacle in the continued scaling of Complementary Metal-oxide Semiconductor (CMOS) microchips in future nano-scale technology generations. Statistical variations between transistors mainly occur due to the random number and position of discrete dopants - chemical spices introduced in the silicon of which the microchips are made to form the structure of the individual transistors. This statistical variability means that circuits built from billions of transistors with individually-unique properties may not perform as well as expected, despite being manufactured in an identical way.

In this work, the threshold-voltage-based BSIM4 compact model is under consideration. A detailed analysis of metamodeling procedure in this particular case has been done. Sobol sensitivity approach is applied for a selected subset of parameters to analyze statistical variability.

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On the N-wave Equations and Soliton Interactions in Two and Three Dimensions

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Several important examples of the N-wave equations are studied [1]. These integrable equations can be linearized by formulation of the inverse scattering as a local Riemann-Hilbert problem (RHP). Several nontrivial reductions are presented. Such reductions can be applied to the generic N-wave equations but mainly the 3- and 4-wave interactions are presented as examples. Their one and two-soliton solutions are derived and their soliton interactions are analyzed. It is shown that additional reductions may lead to new types of soliton solutions. In particular the 4-wave equations with $\mathbb{Z}_2 \times \mathbb{Z}_2$ reduction group allows breather-like solitons. Finally it is demonstrated that RHP with sewing function depending on three variables t, x and y provides some special solutions of the N-wave equations in three dimensions.

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Numerical Study of Gas Temperature Distribution in a Copper Bromide Laser Active Medium

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A numerical model for determining gas temperature distribution in the cross-section of the laser tube of a copper bromide (CuBr) laser is presented. The model is derived on the basis of quasistationary heat conduction equation subject to special mixed boundary conditions. For the first time the real physical conditions, such as the temperature of the reservoirs and other design elements are taken into account. The obtained temperature profile allows to find out specific regions in the active laser medium where the laser generation fails to be effective due to the gas overheating. These results are applicable in research and engineering practice for improving the laser tube design and the overall device performance.

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Web Service Module for Access to g-Lite

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g-Lite is a lightweight grid middleware for grid computing installed on all clusters of the European Grid Infrastructure (EGI). The middleware is partially service-oriented and does not provide well-defined web services for job management. The existing web services in the environment can not be directly used by grid users for building service compositions in EGI. In this article we present a module of well-defined web services for job management in EGI. We describe the architecture of the module and the design of the developed web services. The presented web services are composable and can participate in service compositions (workflows).

An example of usage of the module with tools for service compositions in g-Lite is shown.

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On the Integrability of KdV Hierarchy with Generic Self-consistent Sources

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Non-holonomic deformations of integrable equations of the KdV hierarchy are studied by using the expansions over the so-called "squared solutions" (squared eigenfunctions). Such deformations are equivalent to perturbed models with external (self-consistent) sources. In this regard, the KdV6 equation is viewed as a special perturbation of KdV equation. Applying expansions over the symplectic basis of squared eigenfunctions, the integrability properties of the KdV hierarchy with generic self-consistent sources are analyzed. This allows one to formulate a set of conditions on the perturbation terms that preserve the integrability. The perturbation corrections to the scattering data and to the corresponding action-angle variables are studied. The analysis shows that although many nontrivial solutions of KdV equations with generic self-consistent sources can be obtained by the Inverse Scattering Transform (IST), there are solutions that, in principle, can not be obtained via IST. Examples are considered showing the complete integrability of KdV6 with perturbations that preserve the eigenvalues time-independent. In another type of examples the soliton solutions of the perturbed equations are presented where the perturbed eigenvalue depends explicitly on time. Such equations, however in general, are not completely integrable.

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Mathematical Modeling of the 3D Separated Viscous Fluid Flows

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The 3D homogeneous and density stratified viscous fluid flows around a sphere and a circular cylinder have been investigated by means of the direct numerical simulation on supercomputers and the visualization of the 3D vortex structures in the wake. These flows have been simulated on the basis of the Navier-Stokes equations (NSE) in the Boussinesq approximation. For solving NSE the Splitting on physical factors Method for Incompressible Fluid flows (SMIF) with hybrid explicit finite difference scheme (second-order accuracy in space, minimum scheme viscosity and dispersion, capable for work in wide range of Reynolds (Re) and internal Froude (Fr) numbers and monotonous) has been developed and successfully applied. For the visualization of the 3D vortex structures in the wake the isosurfaces of beta have been drawing, where beta is the imaginary part of the complex-conjugate eigen-values of the velocity gradient tensor. The different transitions in wakes (2D-3D, laminar-turbulent and others) with increasing of Re ($1 < \mathrm{Re} < 500000$) and decreasing of Fr ($0.005 < \mathrm{Fr} < 1000$) have been investigated in details. The classifications of flow regimes have been refined. The unsteady periodical flows at the moderate Re are described through the chain of the basic formation mechanisms of vortices (FMV). The quantitative characteristics of flows obtained by SMIF at moderate Re are in a good agreement with the existing experimental data.

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FETI Solvers for a Non-standard Finite Element Method based on Boundary Integral Operators

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We present efficient Domain Decomposition solvers for a class of non-standard Finite Element Methods. These methods utilize PDE-harmonic trial functions in every element of a polyhedral mesh, and use boundary element techniques locally in order to assemble the finite element stiffness matrices. For these reasons, the terms BEM-based FEM or Trefftz-FEM are sometimes used. In the present talk, we show that Finite Element Tearing and Interconnecting (FETI) methods can be used to solve the resulting linear systems in a quasi-optimal and parallel manner. An important theoretical tool are spectral equivalences between FEM- and BEM-approximated Steklov-Poincaré operators.

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Parametric Time Series Analysis of Daily Air Pollutants of City of Shumen, Bulgaria

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The urban air pollution is one of the main factors determining the ambient air quality, which affects on the human health and the environment. In this paper parametric time series models are obtained for studying the distribution over time of primary pollutants as sulphur and nitrogen oxides, particulate matter and a secondary pollutant ground level ozon in the town of Shumen, Bulgaria. The ARMA and multiple linear regression methods are used to carry out the time series analysis based on daily average data in 2011 and first quarter of 2012. The constructed models are applied for a short-term air pollution forecasting. The results are estimated on the basis of national and European regulation indices. The sources of pollutants in the region and their harmful effects on human health are also discussed.

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Sensitivity Analysis of Ozone Concentration over Territory of Bulgaria

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The US EPA MODELS3 SYSTEM includes MM5 – meteorological preprocessor, SMOKE – emission preprocessor and CMAQ – chemical transport model. TNO emission inventory is used as emission input. The Models-3 "Integrated Process Rate Analysis" option is applied to discriminate the role of different dynamic and chemical processes for the pollution for all SNAP categories. The goal of our studies is to evaluate the influence of the different input parameters as concentration in different SNAP categories under constant meteorological conditions to the output concentrations over Bulgaria. A large number of numerical experiments were carried out, in order to distinguish the contribution of different SNAP categories. Some results from several emission scenarios which make it possible to evaluate the contribution of different SNAP are demonstrated well. The input conditions are varied following the methodology of Sobol-Saltelli. As a result of our computations we present the Sobol coefficients, analyze the results and assess the relative importance of the various parameters, and their interactions.

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Transient Analysis of the Output Process in the $GI/M/1$-type Queue with Finite Buffer

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Queueing systems with finite buffers are widely used for modeling real-life processes occurring in telecommunication, computer networks, manufacturing, transport or logistics. One of the main applications is connected with the operation of the Internet router in which the traffic of incoming/outcoming packets can be described by a proper finite-buffer queueing system.

Theoretical results obtained for different modifications of finite systems (see, e.g., [7]) are mainly restricted to the case of Poisson arrivals and the stationary state of the system (as the time tends to infinity). However, from the observation follows that, e.g., the real stream of packets arriving to the router in the TCP/IP network rather rarely can be described by a Poisson process. Moreover, due to the permanent changing traffic intensity, the investigation of stochastic characteristics of the system in the stationary state has theoretical importance only, since the real system may never reach the stationary state in practice.

In the paper the single-server system of the $GI/M/1$-type with finite buffer is considered, with generally distributed independent interarrival times and exponential service times. In the transient state of the system the output process (called also the departure counting process) $h(t)$ is studied, that at any fixed moment t takes on a random value equal to the number of packets completely served before t. In the case of infinite-buffer systems, in particular for different server vacation policies, the output process was studied in [1], [2] and [3]. The analysis for the compound Poisson arrival process can be found in [4].

A system of integral equations for the conditional distribution of $h(t)$, i.e., for the expressions

$$H_n(t,m) = \mathbf{P}\{h(t) = m \,|\, X(0) = n\}, \quad t > 0,\, m \geq 0,\, 0 \leq n \leq N,$$

is derived, where $X(0)$ stands for the number of packets resent in the system at the opening and N is the total system capacity. Using the approach introduced in [5] (see also [6]) the solution of the corresponding system written down for double transforms of $H_n(t,m)$ is obtained, and stated explicitly using the sequence called potential which is defined by means of input distributions of the system. Sample numerical examples are attached as well.

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Propagation Behavior of Ultrashort Light Pulses

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Spatiotemporal dynamics of ultrashort light pulses is investigated in three basic cases based on (3+1)D nonlinear Schrödinger equation, (3+1)D nonlinear envelope equation, and (3+1)D nonlinear wave equation. The physical models within the specified cases are presented. Some distinct propagations regimes are found. Pulse compression and generation of high-intensity ultrashort light pulses down to single-cycle regime is investigated. We use splitting method in Fourier space for treating the problem.

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Algebraic Multilevel Methods Based on Domain Decomposition

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Multilevel (multigrid) and domain decomposition methods are widely used as efficient solvers for systems of linear algebraic equations arising from finite element discretization of partial differential equations (PDE).

In this talk hybrid multilevel methods are considered that result from recursive application of a two-level block factorization on a hierarchy of overlapping subdomains. The subdomain matrices at the coarse level are constructed from those on the fine level subject to a partition of unity. Once the coarse degrees of freedom have been selected, the local coarse grid matrices are defined by the subdomain Schur complements, and, via assembling yield a global coarse-grid operator.

Several possibilities of exploiting this idea in the construction of multilevel methods are discussed. Similarities and differences to classical multigrid and domain decomposition methods are addressed. Local condition number estimates for the involved additive Schur complement approximation are derived and a two-level convergence analysis is presented. The robustness of the related multilevel algorithms with respect to jumps in the coefficients of the PDE is demonstrated by numerical results.

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Global Existence of the Solutions to Boussinesq Paradigm Equation with Supercritical Initial Energy

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We consider the Cauchy problem to Boussinesq Paradigm Equation

$$\begin{aligned}
\frac{\partial^2 u}{\partial t^2} = \Delta u + \beta_1 \Delta \frac{\partial^2 u}{\partial t^2} - \beta_2 \Delta^2 u + \Delta f(u), \quad x \in \mathbb{R}^n, \quad t \in \mathbb{R}^+, \\
u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), \quad x \in \mathbb{R}^n, \\
f(u) = \alpha |u|^p, \ p > 1, \qquad \alpha, \beta_2 = \text{const} \neq 0, \quad \beta_1 = \text{const},
\end{aligned}$$

which arises in the modeling of surface waves in shallow waters. By a series of numerical experiments we show that the global solvability of the weak solutions with supercritical initial energy $E(0) > d$ depends not only on the sign of the so-called Nehari functional $I(u_0)$ but also essentially on the initial velocity u_1. This heuristic conclusion is theoretically confirmed introducing new energy functional $K(u)$ which depends on u_0 and u_1 and preserves its sign under the flow of the equation. Existence of global weak solutions with high initial energy is proved and their asymptotic behavior for $t \to \infty$ is investigated.

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Numerical Simulation of Flows in Highly Heterogeneous Porous Media

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We shall present an overview of some approximation strategies for numerical solution of flows in highly heterogeneous porous media and robust solution methods for the resulting algebraic system. Our main goal is derivation of numerical methods that work well for both, Darcy and Brinkman equations, and could be used either (1) as a stand alone numerical upscaling procedure or (2) as robust (with respect to the high contrast of the porous media) iterative solvers for the finite element approximation on a fine-mesh spatial scale. The preconditioner is based on an overlapping domain decomposition technique. The robustness with respect to the contrast is achieved via special construction of a coarse grid space that includes patched together eigenfunctions corresponding to the smallest eigenvalues of properly weighted local spectral problems. This approach has a natural abstract framework which we shall discuss as well.

The main target of our applications are numerical upscaling and simulation of fluid flows in highly heterogeneous media modeled by Brinkman, Darcy, and steady-state Richards' equation (including Haverkamp and van Genuchten relations for the relative permeability).

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A Numerical Approach to the Non-convex Dynamic Problem of Pipeline-soil Interaction under Environmental effects

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A numerical approach for a problem arising in Civil and Environmental Engineering is presented. This problem concerns the dynamic soil-pipeline interaction, when unilateral contact conditions due to tensionless and elastoplastic softening/fracturing behaviour of the soil as well as due to gapping caused by earthquake excitations are taken into account. Moreover, soil-capacity degradation due to environmental effects are taken into account. The mathematical formulation of such dynamic elastoplasticity problems [1] leads to a system of partial differential equations with equality domain and inequality boundary conditions [2]. The proposed numerical approach is based on a double discretization, in space and time, and on mathematical programming methods. First, in space the finite element method (FEM) is used for the simulation of the pipeline and the unilateral contact interface, in combination with the boundary element method (BEM) for the soil simulation. Concepts of the non-convex analysis are used [3]. A dynamic hemivariational inequality approach is used for a stability criterion concerning the numerical scheme. Next, with the aid of Laplace transform, the equality problem conditions are transformed to convolutional ones involving as unknowns the unilateral quantities only. So the number of unknowns is significantly reduced. Then a marching-time approach is applied and a non-convex linear complementarity problem is solved in each time-step [3,4]. Finally, a real case of Geotechnical Engineering, where a soil part appears capacity degradation due to environmental effects, is numerically investigated

Keywords: Civil and Environmental Engineering, Soil-Structure Interaction, Numerical Methods, Optimization Algorithms

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Improving the Efficiency of Parallel Alternating Directions Algorithm for Time Dependent Problems

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We consider the time dependent Stokes equation on a finite time interval and on a uniform rectangular mesh, written in terms of velocity and pressure. A parallel algorithm based on a direction splitting approach is implemented. Our work is motivated by the need to improve the parallel efficiency of our supercomputer implementation of the parallel algorithm.

We are targeting the IBM Blue Gene/P massively parallel computer, which features a 3D torus interconnect. We study the impact of the domain partitioning on the performance of the considered parallel algorithm for solving the time dependent Stokes equation. Here, different parallel partitioning strategies are given special attention. The implementation is tested on the IBM Blue Gene/P and the presented results from numerical tests confirm that decreasing the communication time we obtain better scalability of the algorithm.

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Balanced Semi-Coarsening AMLI Preconditioning of Higher Order Elliptic Problems

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RICAM, Linz, Austria

In this talk higher order finite element systems of equations evoked by the scalar and the coupled vector valued elliptic boundary value problems are considered. The finite element discretizations have been performed for both bicubic Lagrangian and bicubic Hermitian elements. The properties of the hierarchical two-level splittings of the finite element spaces that have been induced by the semi-coarsening mesh refinement are studied through analyzing the behavior of the constant in the strengthened Cauchy-Bunyakowsky-Schwarz (CBS) inequality which is a quality measure for hierarchical two-level splittings of the FEM stiffness matrices. A comparison of the CBS constants for the different types of elements is further accomplished. Finally, the consequences of the comparative analysis for meeting sufficient optimality conditions for Algebraic Multilevel Iteration (AMPLI) algorithms are discussed.

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On Nonstandard Finite Difference Schemes in Biosciences

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Biological processes that arise in science are very complex. A lot of effort has been and is being made to build differential models that aim at elucidating these phenomena. However, these models cannot be completely solved by analytic techniques. Consequently, reliable numerical methods are of fundamental importance in gaining some useful insights on the solutions of the differential equations. Of paramount importance for the involved dynamical systems is the design of numerical methods that replicate their underlying dynamics such as the positivity of solutions, the dissipativity of the systems, the conservation laws, the stability of equilibria, etc. This talk is based on the Nonstandard Finite Difference (NSFD) method, which has shown great potential in producing numerical schemes that are dynamically consistent with the properties of continuous models. In a first step, we give an overview of our previous works on reliable NSFD schemes for some compartmental epidemiological models of human diseases, including vector-borne ones. In particular, we establish the numerical counterpart of Kamgang-Sallet's matrix decomposition method and use it to prove that, for each NSFD scheme, the disease-free equilibrium is globally asymptotically stable under the very same threshold conditions on the basic reproduction number, as for the continuous models. Secondly, the study is extended to advection reaction diffusion equations for the modeling of diseases on the one hand and of enzyme kinetics on the other hand. In the process, concerns pertaining to some types of diffusion are raised. Finally, the investigation is taken one step further, namely to the design of NSFD schemes for a class of Volterra integral equations or integro-differential equations that arise in the modeling of the dynamics of disease transmission. The emphasis of the presentation is on both the theoretical analysis of the NSFD schemes and the reliability of the resulting simulations.

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Traveling Waves, Impulses and Diffusion Chaos in Excitable Media

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In the present work it is shown, that the FitzHugh-Nagumo type system of partial differential equations with fixed parameters can have an infinite number of different stable wave solutions, traveling along the space axis with arbitrary speeds, and also traveling impulses and an infinite number of different states of spatiotemporal (diffusion) chaos. Those solutions are generated by cascades of bifurcations of cycles and singular attractors according to the FSM theory (Feigenbaum-Sharkovskii-Magnitskii) in the three-dimensional system of Ordinary Differential Equations, to which the FitzHugh-Nagumo type system of partial differential equations with self-similar change of variables can be reduced.

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Twisted Edwards Curves Integer Factorization on GPU Cluster

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The problem of integer factorization is of high practical importance because of the widespread use of public key cryptosystems for encryption and authentication of Internet connections. In this paper we describe our implementation of the elliptic curve method of integer factorization based on the twisted Edwards curves. The implementation is realized on an NVIDIA GPU-based cluster using the CUDA technology. Also we report the results of our experiments regarding the performance of our implementation.

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On Semi-coarsening Multilevel Preconditioning of Higher Order FEM Systems

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While a huge amount of papers are dealing with robust multilevel methods and algorithms for linear FEM elliptic systems, the related higher order FEM problems are much less studied. It is well known that the standard hierarchical basis two-level splittings deteriorate for strongly anisotropic quadratic FEM problems. First robust multilevel preconditioners for biquadratic anisotropic FEM elliptic problems were recently developed. We study the behavior of the constant in the strengthened CBS inequality for semi-coarsening mesh refinement which is a quality measure for hierarchical two-level splittings of the considered biquadratic FEM stiffness matrices. Some new results for the case of balanced semi-coarsening is of a particular interest. The presented theoretical estimates are supported by numerically computed CBS constants for a rich set of parameters (coarsening factor and anisotropy ratio). Combining the proven uniform estimates with the theory of the Algebraic MultiLevel Iteration (AMLI) methods we obtain optimal order multilevel algorithms whose total computational cost is proportional to the size of the discrete problem with a proportionality constant independent of the anisotropy ratio. The provided comparative analysis of the pure and balanced semi-coarsening algorithms addresses both computational complexity and parallel implementation issues.

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A Direction Splitting Algorithm for Flow Problems in Complex/Moving Geometries

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An extension of the direction splitting method for the incompressible Navier-Stokes equations proposed in [1], to flow problems in complex, possibly time dependent geometries will be presented. The idea stems from the idea of the fictitious domain/penalty methods for flows in complex geometry. In our case, the velocity boundary conditions on the domain boundary are approximated with a second-order of accuracy while the pressure subproblem is harmonically extended in a fictitious domain such that the overall domain of the problem is of a simple rectangular/parallelepiped shape.

The new technique is still unconditionally stable for the Stokes problem and retains the same convergence rate in both, time and space, as the Crank-Nicolson scheme. A key advantage of this approach is that the algorithm has a very impressive parallel performance since it requires the solution of one-dimensional problems only, which can be performed very efficiently in parallel by a domain-decomposition Schur complement approach. Numerical results illustrating the convergence of the scheme in space and time will be presented. Finally, the implementation of the scheme for particulate flows will be discussed and some validation results for such flows will be presented.

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Surface Equations and Singularity at a Triple Contact Line for a Deformable Body

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At a solid-fluid-fluid triple contact line (e.g., solid-liquid-gas), the action of the fluid-fluid surface tension on the elastic solid is a force concentrated on this line, which produces a singularity: the classical theory of elasticity predicts an infinite displacement at this line. We propose a new solution of this problem, with a finite displacement at the contact line, based on surface thermodynamic properties. We first review surface thermodynamics on a rigorous basis (for a deformable body) and then obtain the general equations at surfaces and contact lines.

We first refine the 'dividing surface' concept of Gibbs, by defining the 'ideal transformation' (displacement of the material points, extrapolated up to the dividing surface), determine the thermodynamic variables of state of the surface, define the 'surface stress' tensor and write the surface thermodynamics equations.

We then obtain the general mechanical equation of the surface: it is similar to the classical Cauchy equation for the volume (but here, with the surface stress). It involves the covariant divergence of the surface stress, and its normal component is a generalization of Laplace equation (related to the principal curvatures of the surface).

At a triple contact line, although the components of the (volume) stress tensor do not belong to the Sobolev space H_1, Green's formula may be applied, which leads to two new equations. The first one represents the equilibrium of the forces which act on the contact line considered as fixed on the body (these forces are the surface stresses). The second one expresses the equilibrium relative to the motion of this line (with its attached singularity) with respect to the body, and involves the discontinuity of the deformation of the body surface, when crossing the contact line. It leads to a strong modification of the classical capillary Young's equation.

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An Efficient Highly Parallel Implementation of a Large Air Pollution Model on an IBM Blue Gene Supercomputer

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Z. Zlatev

NERI, Roskilde, Denmark

In this talk we discuss the efficient distributed-memory parallelization strategy of the Unified Danish Eulerian Model (UNI-DEM). We apply an improved partitioning strategy to the spatial domain in order to get more parallel tasks (based on the larger number of subdomains) with less communications between them (due to optimization of the overlapping area when the advection-diffusion problem is solved numerically). This kind of "square-like" partitioning allows us not only to increase significantly the number of potential MPI tasks, but also to reduce the local memory requirements per task, which is critical for the implementatation of the highest-resolution versions of UNI-DEM on the particular IBM BlueGene/P platform – our target hardware. We will show by experiments that our new parallel implementation can use rather efficiently the resources of this powerful supercomputer, the largest in Bulgaria, up to its full capacity.

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Numerical Simulation of Nonlinear Electromagnetic Wave Propagation in Nematic Liquid Crystal Cells

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In the current work, the nonlinear problem of electromagnetic wave propagation in a Nematic Liquid Crystal cell is solved numerically. The liquid crystal (LC) is sandwiched between two glass layers of finite thickness and a linearly polarized beam is obliquely incident to the cell. The dielectric properties of Nematic Liquid crystals depend on the twist angle of the directors. When the excitation beam enters the cell, and providing the incident intensity is above the Fréedericksz threshold, the directors reorient changing the LC relative permittivity tensor. In turn, this affects the beam propagation. The electromagnetic field is modeled by the time-harmonic Maxwell equations whereas the director field is governed by a nonlinear ordinary differential equation. Our solution method is iterative, consistently taking into account this interaction between the excitation beam and the director field. The Maxwell equations are solved employing the Mode-Matching Technique. The solution of the nonlinear differential equation for director field is obtained with the aid of a finite difference scheme.

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AiG – Agent-based Resource Management System for the Grid

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Recently we have completed the first version of an agent-based resource management system for the Grid (the AiG system). This system is providing high-level

intelligent infrastructure for management of resources in an open Grid. In the proposed approach, resources are contracted on the basis of a simple economic model instantiated in ontologies and functionalized through semantic data processing in agreement negotiations. The aim of the presentation will be to present the main tenets of the system, illustrated by examples of a working prototype.

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Numerical Modeling of Wave Propagation in Liquid Crystals using a Mode-matching Approach

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Liquid crystals are quite intriguing materials widely used in optics for a vast range of applications including displays, tunable filters, optical modulators, imaging and recording, temperature and stress sensors, etc. They exhibit tunable characteristics as their electric or magnetic properties can be effectively tuned with a biased electric or magnetic field. Their material tensors are strongly depended on the orientation of the molecules (called directors) whose tilt angle depends on the intensity of the bias field. In cholesteric liquid crystals, the orientation of crystal molecules follow a periodic pattern in the form of a helix. In our presentation in Varna, we will present the underlined mathematical approach that was used in the full-wave characterization of these materials and illustrate its accuracy and effectiveness by comparing results and computational statistics. The reflection and transmission spectra from right-handed cholesteric liquid crystals are computed in the visible spectrum for a linearly, circularly, or elliptically polarized incident plane wave at oblique incidence. The liquid crystal cell is sandwiched between dielectric layers of certain thickness and refractive index. The underlined formulation is based on modal analysis of the governing field expressions in the dielectric and liquid-crystal regions. Due to the anisotropic nature of the dielectric tensor, the ordinary and extraordinary waves inside the cholesteric liquid crystal are coupled and cannot be solved separately.

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Sinuous Instability of a Viscous Capillary Jet

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As shown by Rayleigh (1978) [4] an isolated liquid jet is unstable to axisymmetrical disturbances only. However when the liquid jet interacts by surrounding immiscible fluid a sinusoidal mode of instability appears, that deflects the jet axis from its rectilinear form. It should be mentioned that from a theoretical point of view the sinuous instability is less analyzed. The first linear analysis of the instability of a nonviscous jet was performed by Weber (1931) [5]. Similar solution was proposed by Debye and Daen (1959) [1]. Both analyses are based on the Euler equations of motion written in cylindrical coordinates. Using the latter Martinon (1983) [3] studied higher non-symmetrical modes of instability.

The full 3D-Navier-Stokes equations should be used when the viscosity of the jet is taken into account. In Entov and Yarin (1984) [2] the sinuous instability of a viscous jet was studied by using the so-called "quasy-one-dimensional equations" derived for thin jet as an averaged form of Navier-Stokes equations.

The aim of the present paper is to derive a dispersion equation for the sinusoidal disturbances propagating along a viscous jet, flowing down a nonviscous fluid. An asymptotic analysis of this equation is performed for small wave numbers and large Reynolds numbers. Additionally a numerical analysis of the full version of the dispersion equation is proposed and illustrated.

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Applications of Fitzpatrick Functions for Solving Optimization Problems

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We introduce with the present work applications of Fitzpatrick functions, using some their specific properties as the maximal monotonicity, proper, convex and lower semi-continuity, for solving optimization problems and consider some of their interpretations. We present also an iterative procedure.

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Spatiotemporal Chaos due to Spiral Waves Core Expansion and Its Elimination in Excitable Media

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In the framework of the Fitzhugh-Nagumo kinetics and the oscillatory recovery in excitable media, we present a new type of meandering of the spiral waves, which leads to spiral break up and spatiotemporal chaos. The tip of the spiral follows an outward spiral-like trajectory and the spiral core expands in time. This type of destabilization of simple rotation is attributed to the effects of curvature and the wave-fronts interactions in the case of oscillatory damped recovery to the rest state. This model offers a new route to and caricature for cardiac fibrillation, and when we apply the feedback resonant drift method, for defibrillation all wave activity gets eliminated at the unexcitable boundaries.

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Smart Shells Structures, Modeling and Control

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The piezoelectric effect is understood as the linear electromechanical interaction between the mechanical and the electrical state in crystalline materials with no inversion symmetry. Starting from the constitutive equations of a piezoelectric material, we construct a Koiter shell model coupled with a piezoelectric actuator. After a mechanical interpretation of the effect of the actuator we prove a result of existence and uniqueness of the coupled model. We then discuss controllability results of Galerkin approximations of the model. We use finite element method for the numerical approximation of the coupled model. The numerical tests carried out show the effectiveness of different control algorithms such as the projected Riccati algorithm. This efficiency depends strongly on the position and the shape of the actuator. We then present an algorithm for shape optimization based on the minimization of a criterion. The latter depends on the geometry of the actuator. The gradient of this criterion is calculated using techniques of domain derivation.

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A Third-order Computational Method for Numerical Fluxes to Guarantee Nonnegative Difference Coefficients for Advection-diffusion Equations in a Semi-conservative Form

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According to Godunov theorem for numerical calculations of advection equations, there exist no higher-order schemes with constant positive difference coefficients in a family of polynomial schemes with an accuracy exceeding the first-order. We propose a third-order computational scheme for numerical fluxes to guarantee the non-negative difference coefficients of resulting finite difference equations for advection-diffusion equations in a semi-conservative form, in which there exist two kinds of numerical fluxes at a cell surface and these tow fluxes are not always coincident in non-uniform velocity fields.

The present scheme is optimized so as to minimize truncation errors for the numerical fluxes while fulfilling the positivity condition of the difference coefficients which are variable depending on the local Courant number and diffusion number. The feature of the present optimized scheme consists in keeping the third-order accuracy anywhere without any numerical flux limiter. We extend the present method into multi-dimensional equations. Numerical experiments for linear and nonlinear advection-diffusion equations were performed and the present scheme applicability to nonlinear Burger's equation was confirmed.

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Resonant Behavior of a Fractional Oscillator with Fluctuating Mass

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Stochastic, ordinary, or partial differential equations are very common tools in far-from-equilibrium systems in natural sciences and engineering, where stochastic fluctuations are necessary for an appropriate description of the phenomenology involved [1]. One of the objects of special attention in this context is the noise-driven fractional oscillator, which is suitable for investigation of physical and biological phenomena with a long memory, e.g. in viscoelastic media [2]. Although the behavior of the fractional oscillator has been investigated in detail, it seems that analysis of the potential consequences of an interplay between the fluctuations of the mass of the oscillator and memory effects is still missing in literature. As mass fluctuations of entities can be encountered in many nonequilibrium cases, e.g., in chemical and biological solutions in which the surrounding medium contains molecules capable of adhering to Brownian particles [3], we examine the phenomenon of stochastic resonance of a fractional oscillator that has a random mass and is subjected to an external periodic force. The colored fluctuations of the oscillator mass are modeled as a dichotomous noise. The viscoelastic type friction kernel with memory is assumed as a power-law function of time. Using the Shapiro-Loginov formula and the Laplace transformation technique, an exact expression of the spectral amplification is derived from the basic model. The main purpose of this work is to demonstrate, based on exact expressions, that stochastic resonance is manifested in the dependence of the spectral amplification upon the colored noise parameters. Influence of the memory exponent on the resonance regimes of the oscillator is also investigated. Particularly, it is established that the critical memory exponent which marks a dynamical transition in the resonant behavior of the system considered strongly depends on the ratio of the period of the external deterministic force to the correlation time of the noise.

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Patch-fitting on Surface Holes

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The problems associated with defining the surface shape of a complex deformable object such as a human figure are considerable. As yet, nobody has devised a system which is flexible enough to cater for the whole range of possible surface shapes of such a figure, while retaining the capability of displaying fine detail on the surface, and being sufficiently economical in use for it to be suitable for producing animations in a commercial situation.

A lot of research focusing on developing schemes to produce surfaces with fine detail has been successfully carried out during the last three decades. However, most of these efforts fell some way short in addressing the problem of branching of surfaces. Our work seeks to address this issue. Investigation of the properties of a variety of methods used to tackle the problem of single or multiple branching on a surface has been carried out. What emerged from this investigation was that there does not exist a completely satisfactory method at present, and that some element of compromise is involved in all approaches adopted to date. Additional investigation was necessary in order to derive a new method contributing to further development of complete complex deformable objects.

The method proposed and described in this paper superimposes subdivision-surfaces on top of surfaces containing “holes,” producing completely closed and smooth objects. This provides an additional control over non-rectangular regions of the surface.

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Four-stage Computational Technology with Adaptive Numerical Methods for Computational Aerodynamics

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Despite progress in supercomputer, the computational modeling of viscous flow around complex aerodynamic configurations is still very expensive and often does not give an acceptable accuracy. In our talk, we demonstrate several advances in computational algorithms and numerical analysis to build the four-stage technology with adaptive choice of objectives. We focus on time-dependent Navier-Stokes equations for gas, because they provide almost all typical difficulties.

The first stage "Input and Preliminary Solving" is to solve the Problem by finite element method on the almost uniform grid. The second stage is called "the Prepack" in connection with its result. At first, problematic areas with large derivatives (shock waves, boundary layers, etc.) are found out in solution of previous stage. Then triangulation is modified: became condensed in the problematic areas and sparse in the areas with smooth behavior of the approximate solutions. Finally, the equations are solved again at the modified triangulation.

The third stage "Improving Parameters of Model" consists in refining parameters of mathematical model by assimilation of additional physical data. For this, we construct and solve the Problems which are adjoint to the initial differential Problem in the sense of some linear functionals. These functionals are known for "true" solution and equal some real physical data. Therefore we vary some parameters of model to get namely these values of functionals.

The fourth stage "The Goal-Oriented Task" consists of computing some important parameters like drag, lift, etc. At first, we solve the adjoint Problem in order to find a "sensitivity functions" for these desired parameters. These functions give information for further reconstruction of triangulation, which we do on the next step. Finally, we again solve the required Problem at new triangulation. Each stage will be illustrated by some numerical examples and improving accuracy.

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Comparison of Some Approximation Schemes for Convective Terms for Solving Gas Flow past a Square in a Microchannel

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Rapidly emerging micro-electro-mechanical devices create new potential microfluidic applications. A simulation of an internal and external gas flows is important for their design. For small Knudsen number Kn < 0.1 (Kn $= l_0/L$, where l_0 is the mean free path of the gas molecules and L is the characteristic length), a continuum approach based on modified Navier-Stokes-Fourier or extended hydrodynamic continuum models with corresponding velocity-slip and temperature-jump boundary conditions is still applicable and, respectively, preferable. We restrict ourself to the use of Navier-Stokes-Fourier continuum model. A development of the algorithm to solve specific class of a problems is closely related to numerical schemes used for approximation of equations terms. Higher-order approximation schemes can reduce the number of mesh nodes and respectively computational time, but it is possible to obtain physical unrealistic results. In this paper we study influence of some approximation schemes for convective terms over the space steps. It is compared upwind, central difference, hybrid differencing and some total variation diminishing (TVD) schemes. A test case is gas flow past a square in a microchannel, available in a literature.

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Parametric Resonance of a Harmonic Oscillator with Fluctuating Mass

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The influence of random mass perturbation on a dynamical system is a problem of interest in various fields of science and engineering. Many physical cases are concerned by such situations. Random mass or volume variation of entities can be encountered in various nonequilibrium cases: ion-ion reaction, dust agregation, electrodeposition, film deposition, granual flow, even stock markets, to name of few [1]. Recently, inspired by the fact that the harmonic oscillator is the simplest paradigmatic model of different phenomena in nature, the authors of Ref. [2] have investigated the dynamical behavior of harmonic oscillator with according to dichotomous noise fluctuating mass. However, in this work by analysis of results the authors have confined themselves to investigating very small noise amplitude, and thus the possible energetic instability or stochastic parametric resonance is not considered. Thus motivated, we explore the problem of colored noise-induced energetic instability in a harmonic oscillator. The colored fluctuations of the oscillator mass are modeled as a three-level Markovian telegraph noise. Using Shapiro-Loginov formula, the exact expressions of second order statistic moments of the output signal have been calculated. The main purpose of this work is to demonstrate, based on exact expressions, that stochastic parametric resonance is manifested in the dependence of the variance of output signal upon the noise amplitude. The nonmonotonic dependence of the variance and also the conditions for the appearance of energetic instability are analyzed. Our main result is that for harmonic oscillator colored fluctuations of the mass can cause noise correlation-time-induced transitions from energetic stability to instability as well as in the opposite directions. Furthermore, the transition is found to be reentrant, e.g., if the damping coefficient is lower than certain threshold value, then the energetic instability appears above a critical value of the noise correlation time, but disappears again through a reentrant transition to the energetically stable state at higher value of the noise correlation time.

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Influence of Different Boundary Conditions on the Dynamic Behavior of Cracked Magnetoelectroelastic Materials

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Magnetoelectroelastic material with a finite crack is considered. The crack is subjected to an anti-plane mechanical and in-plane electric and magnetic load. The fundamental solutions of the coupled system of the governing equations are derived in a closed form by the Radon transform. They are implemented in a non-hypersingular traction boundary integral equation method (BIEM).

A program code in Fortran, based on the BIEM, is created. Validation studies show the accuracy of the proposed scheme by comparing the results with the available data for impermeable and permeable cracks. Numerical examples display the dependence of the dynamic stress intensity factor on the normalized frequency for different boundary conditions at the crack faces.

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Chaotic Cryptographic Scheme and its Randomness Evaluation

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Cryptography is the essential in data hiding applications and digital transmissions. In recent years, a multifarious of chaotic cryptographic schemes have been proposed. In this paper we propose a new cryptographic scheme based on the Lorenz chaos attractor and 32 bit bent-function. We evaluated the keystream generated by the scheme with batteries of the NIST statistical tests. In addition we applied number of statistical analysis by calculating the histograms, the correlations of two adjacent pixels, the numbers of pixels change rate, the unified average changing intensity and the mean absolute error provided to images encrypted with the proposed system. The results of the analysis show that the new cryptographic scheme ensures a secure way for sending the digital data and real-time image encryption.

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Mathematical Problems in the Full Coupled Theory of Poroelasticity

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Porous materials play an important role in many branches of engineering, e.g., the petroleum industry, chemical engineering, geomechanics, and, in recent years, biomechanics. This paper concerns with the dynamical full coupled theory of poroelasticity for solid with double porosity. This theory is straightforward generalization

of the earlier proposed theories of consolidation with double porosity. In this paper the basic boundary value problems (BVPs) of steady vibrations of the full coupled linear theory of poroelasticity for solid with double porosity are investigated, some basic results of the classical theories of elasticity are generalized and the following results are obtained: the fundamental solution of the system of equations of steady vibrations is constructed by means of elementary functions, the Green's formulae in the considered theory are obtained, the uniqueness theorems of the basic BVPs are proved, the representation of Galerkin type solution is obtained and the completeness of this solution is established, the formulae of integral representations of regular vector and regular (classical) solutions are obtained, the basic properties of the surface and volume potentials and singular integral operators are established, the existence theorems for the basic BVPs are proved by means of the boundary integral method and the theory of singular integral equations.

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Operational Pollution Forecast for the Region of Bulgaria

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An operational prototype of the Integrated Bulgarian Chemical Weather Forecasting and Information System is presented. This version of the system is limited to relatively low resolution (10 km) but covers all Bulgaria. It is based on the US EPA Models-3 System (MM5, SMOKE and CMAQ). The meteorological input to the system is the Bulgarian operational numerical weather forecast. The boundary conditions are taken by the Greek system (Aristotle University of Thessaloniki). The System automatically runs twice a day (00 and 12 UTC) and produces 48-hour forecast. The part of the results of each System's run is post-processed in a way to be visualized and uploaded to a respective web site. In the paper, description of the System is given together with a demonstration of its products. In addition highlights of Systems upgrade will be given.

Key words: Air Quality Modeling, Pollution Forecast, Chemical Weather

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Parameter Estimation in Dynamical Systems with Delays

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Parameter estimation has proven to be a key aspect of using dynamical systems models to gain information about observed processes. Discrete and distributed time delays as well as cumulative effects arise naturally in many biological, sociological, and physical applications. These processes are also commonly nonlinear, and thus addressing mathematical issues concerning parameter estimation with such systems is nontrivial. We discuss here results concerning these issues and point to some future directions.

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Modeling of Wake Vortex Safety in Aviation

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Generation and evolution of wake vortices past aircraft and ground-based structures by means of mathematical modeling methods and revelation of impact of different conditions on intensity, evolution and lifetime of wake vortices are considered. The presence of wake vortices past different objects limits the airspace capacity and flight safety level for aircraft of different purposes. A mathematical model simulating the process of near wake generation past bodies of different shapes, as well as the wake evolution after rolling-up into wake vortices (far wake) will be developed by means of the Method of Discrete Vortices. Far wake evolution is determined by its complex interaction with the atmosphere and ground boundary layer. The main factors that are supposed to take into account are: wind and ambient turbulence 3D-distributions, temperature stratification of the atmosphere, wind shear, as well as some others which effects will be manifested as considerable during the investigation. The ground boundary layer effects on wake vortex evolution are substantial at low flight altitudes and are determined through the boundary layer separation. The separation leads to changes in the wake vortex altitude and intensity.

This work was supported by the Scientific Research Foundation by the Bulgarian Ministry of Education, Youth, and Science under Project DO 02-338/008, and by the Russian Foundation for Basic Research under Project 10-07-00381.

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On the Gravitational Interaction of Light

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The light can be source of gravitational waves belonging to the lager class of exact solutions of Einstein field equations which are invariant for a non-Abelian 2-dimensional Lie algebra of Killing fields. In the would be quantum theory of gravity these gravitational waves will correspond to spin-1 massless particles. Thus, a solution of the old problem risen by Tolman, Ehrenfest, Podolsky and Wheeler, concerning the gravitational interaction of light, is naturally proposed.

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Comparison of Surface Subdivision Algorithms for Polyhedral Meshes

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Subdivision algorithms are one of the most successful modern techniques for modeling smooth surfaces of arbitrary topology. The applications are in various fields: Computer Aided Geometric Design, computer graphics, solid modeling, computer game software, computer animation, aerospace and automotive design, industrial design, architecture, medicine, etc. One of the promising applications is in solving problems that arise in applied physics. For example, subdivision surfaces can be used in simulation of thin shells and plates.

We present a new program package for interactive implementation and 3D visualization of three fundamental algorithms for surface subdivision based on arbitrary polyhedral meshes. Namely, these are Doo-Sabin algorithm, Catmull-Clark

algorithm, and Peters-Reif algorithm. Our work and contributions are in the field of experimental algorithmics and algorithm engineering.

We have chosen OpenGL and Qt graphics libraries as our main implementation and visualization tools. OpenGL is probably the most used in industry application programming interfaces for computer graphics. This is due to its wide accessibility and compatibility with different operating systems and computer platforms.

Our program analyzes the validity of the loaded mesh and proceeds in case it is a valid polyhedral mesh. The program can be used for experiments using the client's data sets. The latter allows the clients to test and compare the results from implementation of the three algorithms on arbitrary polyhedral meshes. The program has also an option for creating new polyhedral meshes.

We have performed extensive experimentation with our package. We have compared the performance of the three algorithms based on different criteria and using meshes of increasing complexity. The experimental results are presented and analyzed.

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Computer Simulation of RF Liver Ablation on an MRI Scan Data

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Radio-frequency (RF) ablation is a low invasive technique for treatment of liver tumors. An RF-probe is inserted in the patient's liver and a ground pad is applied to the skin. Then the tumor is heated with RF current. The heat causes the death of tumor cells. We use finite element method (FEM) to simulate and analyze various aspects of the procedure.

A 3D image of the patient's liver is obtained from magnetic resonance imaging (MRI) scan. The image is segmented into different regions – liver, blood vessels, bones, etc. A multi domain FEM mesh is generated, after adding the geometry for the RF-probe and the ground pad. The related mesh is strongly unstructured.

The complexity of the domain leads to big meshes. We discretize and solve the problem on a parallel computer using MPI for the parallelization. The presented numerical tests are performed on IBM Blue Gene/P machine at BGSC. The parallel efficiency of the incorporated BoomerAMG solver is demonstrated as well.

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Position Sensitive Fission Detection for Prompt Fission Neutron Investigation

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The purpose of the present paper was to report the recent results, obtained in development in digital pulse processing mathematics applied to prompt fission neutron data analysis. The prompt fission neutron (PFN) investigation was done using twin ionization chamber (TIC) along with fast neutron time-of-flight detector (ND). Data was taken using digital electronics and digital signal processing technique, already described in previous publications. In this report authors are providing detailed information on data analysis procedure developed for fission fragment (FF) and PFN kinematics and valuable physical information on nuclear fission process, suitable for comparison with modern phenomenological theoretical results. Some automation of data analysis process are discussed in the paper.

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Prompt Fission Neutron TOF Data Analysis

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The prompt neutron emission in spontaneous fission of ^{252}Cf has been investigated applying digital signal electronics along with associated digital signal processing algorithms. A new mathematical approach, applicable to single events, was developed for prompt fission neutron (PFN) time-of-flight distribution unfolding. The main goal was to understand the reasons of a long time existing discrepancy between theoretical calculations and the measurements of prompt fission neutron (PFN) emission dependence on the total kinetic energy (TKE) of fission fragments (FF). Due to ^{252}Cf (sf) reaction is one of the main references for nuclear data the

understanding of PFN emission mechanism is very important both for nuclear fission theory and nuclear data. Experimental data was taken with a twin Frisch-grid ionization chamber and a NE213-equivalent neutron detector in an experimental setup similar to well work of C. Budtz-Jorgensen and H.-H. Knitter. About 107 coincidences between fission fragment (FF) and neutron detector response to prompt fission neutron/gamma detection have been registered. Fission fragment kinetic energy, mass and angular distribution, neutron time-of-flight and pulse shape have been investigated using a 12 bit waveform digitizer. The signal waveforms have been analyzed using digital signal processing algorithms. The main goal of this work was in detailed description of prompt fission neutron treatment.

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